Is dissipativeness \(\equiv\) dissipativeness? \ldots When two theories met

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Abstract—In this paper, we construct a bridge between two apparently disjointed theories, both intending to capture the intuitive concept of dissipativeness, and which have coexisted, evolved and grown independently from each other: Passivity, from control systems theory, and point-dissipativeness, from dynamical systems theory. Recently, in [20] it was introduced a control design method for the global CLF stabilization of affine control systems with point-dissipative free dynamics, via admissible (bounded and regular) feedback controls. Therefore, due to the relationship between both theories, we show the underlying passivity-based ideas of that control design.

I. INTRODUCTION

By the intuitive concept of dissipativeness one understand that a system receives more energy that it gives. The formalization of this concept has lead to the born of two main theories: First, dissipativeness, and in particular passivity, in the context of control theory, developed by Willems, Hill, Moylan, \textit{et al}, on one hand; and second, dissipativeness, and in particular point-dissipativeness, in the context of the theory of dynamical systems, developed by Levinson, Pliess, Hale, \textit{et al}, on the other hand.

The concept of passivity, introduced by Popov in the early 1960’s, together with the famous Kalman-Yakubovich-Popov (KYP) Lemma established a bridge between two apparently disjointed disciplines: The frequency domain methods and the stability analysis. In 1972, Willems formalized in [25] the concept of passivity (and dissipativeness) by introducing the notions of storage function and supply rate. Grosso modo, passivity “is the property that the increase in storage is not larger than the integral amount supplied”, [15], p. 4.

The notion of point-dissipativeness was conceived in 1944 by Levinson, when he studied a periodically forced van der Pol equation, and assumed the infinite as a stable point. A system is said to be point-dissipative if and only if (iff) it has a bounded set into which every orbit eventually enters and remains. In 1966, by a result due to Pliess for finite-dimensional systems (local compactness of the state space), we have that a system is point-dissipative iff it has a compact global attractor (see [9], [10]). The goal of this theory is to show that many of the notions of dynamical systems on locally compact spaces can be adapted to those infinite-dimensional dynamical systems having some type of dissipativeness property. However, the lack of local compactness of the state space is overcome imposing additional conditions, e.g. asymptotically smoothness/compactness, see [10], [12].

To the best of our knowledge, in spite of being known as of 1976 (in [11]) till now, these theories have coexisted, evolved and grown independently from each other. In fact, in [5], p. 256, it is written: “The word dissipative is sometimes used in a different context in the theory of dynamical systems”, sentence alluding explicitly to the second theory.

An important control objective is to render a system globally asymptotically stable (GAS) via admissible (bounded and regular) feedback controls. To achieve this aim, we can assume that the system has some type of dissipativeness property, such that all its solutions tend to a specific set: The global attractor \(K\). The attractor \(K\) can be defined as the maximal compact invariant attractive set, and it contains all the “interesting” dynamics for the system, such as equilibrium points, limit-cycles, strange attractors, \textit{et cetera}.

In general, an exact description of the global attractor \(K\) is difficult (or impossible) to have. Instead, an overestimation of it, given by an absorbing ball (a positively invariant attractive bounded set) \(B\), can be enough for many applications. By choosing a Lyapunov function and applying the Lyapunov stability criteria, one can obtain absorbing balls for many systems. For some chaotic systems (e.g. Lorenz, Chen, smooth Chua), this dissipative property has been investigated, which implies that these systems are point-dissipatives.

Recently, chaos control of chaotic systems has become an active research topic. However, although there have been many attempts to control chaotic systems (see [1] and the references therein), there are few results on the problem of constrained control of chaos. In this respect, in [20] we proposed a control design method for the GAS of affine control systems with point-dissipative free dynamics, that includes some chaotic systems, by using admissible (bounded and regular) feedback controls. The result is based on the control design method developed in [18], [19] (entailed from Artstein and Sontag’s control Lyapunov function (CLF) approach for the GAS of affine systems via admissible feedback controls.

The method was exemplified with the constrained control of chaos of a smooth (cubic) Chua’s system (see [24]).

“In [21], it was proposed a control design method for a class of affine control systems with stable free dynamics: Passive systems with zero-state detectability. It is well known that these systems can be rendered GAS using arbitrarily small controls. If we interpret the flow of a system as a fluid, then these systems can be seen as Newtonian fluids, needing any small external force to begin to flow fluently towards 0. On the other hand, other class of systems need to surpass certain minimum control effort before they become GAS, behaving more or less like a non-Newtonian fluid such as ketchup or “mud”.

*This work was partially supported by project “Dynamical Systems and Stabilization”, PROMEP, SEP, México.
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978-1-4673-5716-6/13/$31.00 ©2013 IEEE 3963