Yamauchi, Takamitsu

Asymptotic property C of the countable direct sum of the integers.

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Summary: It is proved that the countable direct sum of the integers with a proper invariant metric has asymptotic property C, which answers a question posed by A. Dranishnikov and M. Zarichnyi.
ACKNOWLEDGEMENT:
Your review has just been received. Thank you very much!

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- Primary Classification: 54F45
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Review:
Given a metric space $(X,d)$ and a set $U \subset X$, the number
\[
\text{diam}(U) = \sup\{d(x,y) : x, y \in U\}
\]
is called the diameter of the set $U$. If $U, U' \subset X$ then $d(U, U') = \inf\{d(x, x') : x \in U \text{ and } x' \in U'\}$. A family $\mathcal{U} \subset \text{subsets of } X$ is uniformly bounded if $\sup\{\text{diam}(U) : U \in \mathcal{U}\}$ is finite. If $r > 0$ then the family $\mathcal{U}$ is $r$-disjoint if $d(U, U') \geq r$ for any distinct $U, U' \in \mathcal{U}$. A metric space $X$ is said to have asymptotic property $C$ if for any sequence $r_0 < r_1 < \ldots$ of positive reals, there exist $n \in \mathbb{N}$ and uniformly bounded families $\mathcal{U}_1, \ldots, \mathcal{U}_n$ such that each $\mathcal{U}_j$ is $r_j$-disjoint and the family
\[
\bigcup_{j=1}^n \mathcal{U}_j
\]
covers $X$.

The group $\bigoplus_{i=1}^\infty \mathbb{Z} = \{x \in \mathbb{Z}^\mathbb{N} : \{i \in \mathbb{N} : x(i) \neq 0\} \text{ is finite}\}$ is the countable direct sum of integers. Given any points $x, y \in \bigoplus_{i=1}^\infty \mathbb{Z}$, let $d(x, y) = \sum_{i=1}^\infty i|x(i) - y(i)|$; it is proved that the metric space $\left(\bigoplus_{i=1}^\infty \mathbb{Z}, d \right)$ has asymptotic property $C$. 


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Review text:
Given a metric space \((X,d)\) and a set \(U \subset X\), the number \(\text{diam}(U) = \sup \{d(x,y) : x, y \in U\}\) is called the diameter of the set \(U\). If \(U, U' \subset X\) then \(d(U, U') = \inf \{d(x, x') : x \in U \text{ and } x' \in U'\}\). A family \(U\) of subsets of \(X\) is uniformly bounded if \(\sup \{\text{diam}(U) : U \in \mathcal{U}\}\) is finite. If \(r > 0\) then the family \(U\) is called \(r\)-disjoint if \(d(U, U') \geq r\) for any distinct \(U, U' \in \mathcal{U}\). A metric space \(X\) is said to have asymptotic property \(C\) if for any sequence \(r_0 < r_1 < \ldots\) of positive reals, there exist \(n \in \mathbb{N}\) and uniformly bounded families \(\mathcal{U}_1, \ldots, \mathcal{U}_n\) such that each \(\mathcal{U}_j\) is \(r_j\)-disjoint and the family \(\bigcup \{\mathcal{U}_j : j \leq n\}\) covers \(X\).

The group \(\bigoplus_{i=1}^{\infty} \mathbb{Z} = \{x \in \mathbb{Z}^\mathbb{N} : \text{the set } \{i \in \mathbb{N} : x(i) \neq 0\} \text{ is finite}\}\) is called the countable direct sum of integers. Given any points \(x, y \in \bigoplus_{i=1}^{\infty} \mathbb{Z}\), let \(d(x,y) = \sum_{i=1}^{\infty} i \cdot |x(i) - y(i)|\); it is proved that the metric space \((\bigoplus_{i=1}^{\infty} \mathbb{Z}, d)\) has asymptotic property \(C\).