Igor R. Shafarevich: Collected Mathematical Papers

Igor R. Shafarevich

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[Reviewed by Felipe Zaldivar, on 01/4/2016]

This is a softcover reprint of the 1989 edition of the collected mathematical papers of I. R. Shafarevich. It is a selection that includes his research papers published between the years 1943 and 1984, plus three expository papers, one of which is a German version of a well known and polemical paper exposing Shafarevich's extreme version of Platonism in mathematics. According to the list of published papers of Shafarevich between the considered years, the selection is complete and only excludes abstracts, summaries of talks given at conferences, and a few pages written on the occasion of anniversaries of notable Russian mathematicians close to Shafarevich.

A quick look at MatSciNet shows that after 1984, and *not* included in his collected mathematical papers, Shafarevich published nine research papers (two of which were listed as "to appear" in the Bibliography section of the book under review), one important survey paper and two nice expository papers, including one where Shafarevich discusses his very important conjecture on the universal cover of a smooth complex projective variety. They might make a nice supplementary volume.

Overall, this rough count gives 49 research papers and 6 expository papers. This count does not include Shafarevich's several important books, from his influential textbooks on algebraic geometry and number theory to his monograph on algebraic surfaces, written in collaboration with several of his students.

Shafarevich's work can be roughly divided into four main areas: First, his early work in algebraic number theory, the highlights being his solution of the inverse Galois problem for solvable groups, a general reciprocity law for number fields, and solving the class-field tower problem in collaboration with E. Golod. Second, his work on algebra, with emphasis on group cohomology and Lie algebras

over fields of positive characteristic. Third, his work on the arithmetic of elliptic curves, some of it in collaboration with J. Tate, which among other things included the construction of the Shafarevich-Tate group of an elliptic curve, a key arithmetic invariant of elliptic curves. Fourth, his work in algebraic geometry, especially on algebraic surfaces, with emphasis on K3 surfaces.

Among Shafarevich's contributions one may also add his several deep conjectures in arithmetic geometry and algebraic geometry. One example may perhaps illustrate how deep Shafarevich's conjectures reach: In his contribution to the *Proceedings of the International Congress of Mathematicians* in Stockholm (1962) and on pages 290–291 in the book under review, Shafarevich conjectures that there are only a finite number of isomorphism classes of curves of given genus, defined over a fixed number field, and with good reduction away from a finite set of primes. This is a far-reaching generalization of the classical finiteness theorems of Hermite and Minkowski (the number of extensions of number fields of given degree and discriminant is finite). The proof of this conjecture by G. Faltings in 1982 had, as a consequence, the Mordell conjecture on the finiteness of the set of rational points of smooth curves of genus at least 2 defined over a number field.

Perhaps I may allowed to repeat what is a well-known fact: Shafarevich's publications reflect only a fraction of his lasting legacy in mathematics, which among other things includes his almost single-handed creation and nurture of the Russian school of algebraic geometry and arithmetic geometry.

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