On-line Condition-Based Maintenance

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Abstract. The aim of this paper is to propose an adequate condition-based maintenance policy to a gradually deteriorating system using on-line detection algorithms. The originality of this work is due to the fact that the parameters defining the deterioration mode can be unknown. The main purpose is to estimate these unknown parameters in order to propose an adoptive condition based maintenance policy and above all to optimise a global cost criterion.

Keywords. IWSM, condition-based maintenance, on-line change detection, preventive maintenance.

1 INTRODUCTION

This paper proposes to develop a maintenance decision rule for a deteriorating system with several modes of degradation when on-line information on the deterioration is available. The proposed on-line parametric maintenance decision rule is optimised in order to minimise an average maintenance cost criterion.

In this paper, we deal with a system which nominally deteriorates according to a given known mode and for which the mean deteriorating rate increases suddenly at an unknown time. The time of change of the degradation mode is called the change time. An adaptive maintenance policy based on on-line change detection procedures is proposed in [4,5] when the parameters of the accelerated mode are known. Throughout this paper the parameters of the accelerated mode are no longer known and the aim is to propose a condition-based maintenance policy based on the estimates of these parameters.

When through an inspection a failure is discovered, a corrective maintenance operation immediately replaces the failed system by a new one. The preventive maintenance actions are performed in order to avoid a failure occurrence and the resulting period of inactivity of the system (i.e. time interval between the instant of failure and the following inspection). The preventive maintenance action takes place when the system's deterioration level exceeds a predetermined threshold known as the preventive threshold.

The global maintenance cost depends on the choice of the inter-inspection times and the value of the preventive threshold. For example, in the case of costly inspections it is not worthwhile to inspect often the system. But if the system is scarcely inspected, the risk of missing a failure occurrence increases. In [6] or [3] and [2] authors propose condition-based inspection/replacement and continuous monitoring replacement policies for a single-mode deteriorating system. In those previous works, a maintenance cost model is proposed which quantifies the costs of the maintenance strategy and propose a method to find the optimal strategy leading to a balance between monitoring and maintenance efficiency. When the system undergoes a change of mode it seems reasonable to incorporate the on-line information available about the system in the maintenance decision rule. In [9] author propose an adaptive maintenance policy for a continuously monitored system provided that the instant of change of mode is immediately and perfectly detected. In [4,5] the on-line change detection in the framework of the condition based maintenance in the case of known parameters after the change is studied.

In this paper an adaptive maintenance policy based on an embedded on-line change detection algorithm is proposed. The originality is due to the fact that the parameters after the change (i.e. the parameters of the second mode) are supposed to be unknown.

In section 2, the deteriorating system is described. Section 3 is devoted to the presentation of the proposed maintenance decision rule. In section 4, an adequate on-line change detection algorithm is proposed. This algorithm takes into account the fact that the parameters after the change are unknown. A method for the evaluation of the maintenance cost is proposed in section 5. The theoretical results are analysed by numerical implementations in section 6.

2 System description

One considers an observable system subject to accumulation of damage. The system's deterioration level (system state) at time t can be summarised by a scalar random ageing variable X_t [11]. In absence of repair or replacement action, $(X_t)_{t\geq 0}$ is an increasing stochastic process, with initial state $X_0 = 0$. The behaviour of the deterioration process after a time t depends only on the amount of deterioration at this time. When the state of the process reaches a pre-determined threshold, say L, the system is said to be failed. The failure of the system occurs as soon as a defect or an important deterioration is present, even if the system is still functioning. In these conditions it is no longer able to fulfil its mission in acceptable conditions. The threshold L is chosen in respect with the properties of the considered system.
The parameters of the deterioration process $(X_t)_{t\geq 0}$ can suddenly change at an unknown time T_0. This means that the mean deterioration rate suddenly increases from a nominal value to an accelerated rate at time T_0. The first mode corresponds to a nominal mode denoted by M_1 and the accelerated mode is denoted by M_2. In this paper, it is assumed that the deterioration process in mode M_i ($i = 1, 2$) denoted by $(X^i_t)_{t\geq 0}$ is a gamma process i.e. for all $0 \leq s \leq t$, the increment of $(X^i_t)_{t\geq 0}$ between s and t, $Y^i_{s,t} = X^i_t - X^i_s$, follows a gamma probability distribution function with shape-parameter $\alpha_i.(t-s)$ and scale parameter β_i. This probability distribution function can be written as follows:

$$f_{\alpha_i(t-s),\beta_i}(y) = \frac{1}{\Gamma(\alpha_i(t-s))} \cdot \frac{y^{\alpha_i(t-s)-1} e^{-\frac{y}{\beta_i}}}{\beta_i^{\alpha_i(t-s)}} \mathbf{1}_{\{y\geq 0\}}. \tag{1}$$

The average deterioration speed rate in mode M_i ($i = 1, 2$) is $\alpha_i.\beta_i$ and its variance is $\alpha_i.\beta_i^2$. Since the mode M_2 corresponds to an accelerated mode the parameters are such that $\alpha_2.\beta_2 > \alpha_1.\beta_1$. Note that the gamma process is a positive process with independent increments, hence it is sensible to use this process to describe the deterioration caused by accumulation of wear [10]. Another interest of the gamma process is the existence of an explicit probability distribution function which permits feasible mathematical developments. In difference with [4,5] in this paper we consider that the parameters of the accelerated mode M_2 are unknown.

3 Maintenance policy

It is supposed that the considered deteriorating systems cannot be continuously monitored. For this reason, the deterioration level can only be known at inspection times. We shall denote by $(t_i)_{i\in\mathbb{N}}$ the sequence of the inspection times defined by $t_{i+1} - t_i = \Delta t$ (for $i \in \mathbb{N}$) where Δt is a fixed parameter. When the deterioration level exceeds the threshold L between two inspections, the system continues to deteriorate until the next inspection. In order to avoid the occurrence of a failure, a preventive maintenance action has to take place before the deterioration level exceeds the threshold L. Furthermore, the inter-inspection time Δt has to be carefully chosen in order to be able to replace the system before the failure.
The time of change of degradation mode T_0 is always unknown. Hence, a detection method can be used to identify T_0. The on-line change detection algorithms presented in section 4 is used to detect the change mode time T_0 when the parameters after the change are unknown. The maintenance decision is based on a parametric decision rule according to the change detection result. As in [9], [5] and [4], the preventive maintenance decision is based on two different preventive thresholds corresponding to each of the two possible deterioration modes (nominal or accelerated) of the system. Such maintenance policies are extensions of inspection/replacement structures for single mode deteriorating systems. Let A_{inf} and

A_{sup} such that $A_{\text{inf}} < A_{\text{sup}}$ be the decision thresholds associated to the two "limit" cases corresponding to the single-mode deterioration system. The decision threshold A_{sup} (respectively A_{inf}) is chosen in order to minimise the cost criterion of the nominal (respectively accelerated) single-mode deteriorating system. In the nominal mode the threshold A_{sup} is effective and as soon as the system is supposed to have switched in the accelerated mode then threshold is adapted from A_{sup} to A_{inf}.
The possible decisions which can arise at each inspection time t_i are as follows:

- If $X_{t_i} \geq L$ the system has failed then it is correctively replaced.
- If no change of mode has been detected in the current cycle (the system is supposed to be in nominal degradation mode) and $X_{t_i} \geq A_{\text{sup}}$ the system is preventively replaced.
- If a change of mode is detected at time t_i or has been detected earlier in the current cycle and if $X_{t_i} \geq A_{\text{inf}}$ then the system is preventively replaced.
- In all the other cases, the decision is postponed to time t_{i+1}.

As a consequence of the previous decision rule, if a change of mode is detected at time t_{detect}, the two following scenarios can arise :

- If $X_{t_{\text{detect}}} < A_{\text{inf}}$ then the system is left unchanged and a replacement is performed at time $t_n > t_{\text{detect}}$ such that $X_{t_{n-1}} < A_{\text{inf}} \leq X_{t_n}$.
- If $X_{t_{\text{detect}}} \geq A_{\text{inf}}$ then the system is immediately replaced.

The parameters of the maintenance decision rule are respectively A_{inf}, A_{sup} and the parameter of the detection algorithm.

4 On-line change detection

The on-line change detection algorithms permit to use the on-line available information on the deterioration rate to detect the occurred abrupt change time. These algorithms take into account the information collected through inspections, so they treat with on-line observations (i.e. system state at times $(t_i)_{i\in\mathbb{N}}$). The quality of an on-line change detection algorithm depends on the mean delay to detection and the false alarm rate [1].
The unknown parameters of the accelerated mode are estimated by the maximum likelihood method. To apply this method on the available observed X_{t_i} from the beginning of the current cycle requires a big amount of calculations. In order to reduce the complexity of calculation, it is suggested in [8,7] to use a sliding window and to take into account only data available in this widow. The size of the sliding window should satisfy some conditions in order to have a fixed upper bound for the false alarm rate and to permit the maximisation of the likelihood function. We shall denote by $\Pr_0$ the probability knowing that no change of mode is occurred, $\Pr_{T_0}$ the probability knowing that the change of mode is occurred at T_0. $\mathbb{E}_0$ (resp. $\mathbb{E}_{T_0}$) is the expectation corresponding to the probability $\Pr_0$ (resp. $\Pr_{T_0}$). Let be $S_k^t = \sum_{i=k}^t \log \frac{f_{\theta_2}(Y_i)}{f_{\theta_1}(Y_i)}$, where $Y_i = X_{t_i} - X_{t_{i-1}}$ and $\theta_j = (\alpha_j, \beta_j)$ for $j = 1, 2$. We shall supposed that $\theta_2 \in \Theta$ and $\Theta \subset \mathbb{R}^2$ is a compact e.g. the possible values of α_2 (resp. β_2) are in a bounded interval. The detection algorithm initially proposed by [8] is defined as follows. The change of mode is detected at $\hat{T}$ such that:

$$\hat{T} = \inf\{t \geq \tilde{M} : \max_{t-m_a \leq k < t-m'_a} \sup_{\theta_2 \in \Theta} S_k^t \geq h\}; \tag{2}$$

The constant h is chosen such that

$$\sup_{k\geq 1} \Pr_0(k - m_a \leq \hat{T} \leq k - m'_a) \leq a, \tag{3}$$

where $\liminf_{a\to 0} \frac{m_a}{|\log a|} > \rho^{-1}$, $m'_a = o(|\log a|)$ but $\log m_a = o(\log a)$, when $a \to 0$ where $\rho = \mathbb{E}_{T_0}\big[\log \frac{f_{\theta_2}(X)}{f_{\theta_1}(X)}\big]$ is the Kullback-Leibler distance between f_{θ_2} et f_{θ_1}. The minimal delay m'_a is used to avoid difficulties with the likelihood maximisation. The author in [7] proved that under strong conditions the delay of the detection rule combined with maximum likelihood estimation method satisfies

$$\mathbb{E}_{T_0}(\hat{T} - T_0)^+ \sim \{\Pr_0(\hat{T} \geq T_0)/\rho + o(1)\}|\log a|, \ \forall T_0 \geq 0 \text{ when } a \to 0. \tag{4}$$

5 Evaluation of maintenance policy

To evaluate a maintenance policy one should take into account costs incurred after each maintenance action. Each corrective (respectively preventive) replacement entails a cost C_c (respectively C_p) where $C_p < C_c$. One shall denote by C_i the inspection cost and by C_u the cost per unit of time incurred in the period of unavailability of the system. One shall denote by $N_p(t)$ (respectively $N_c(t)$ and $N_i(t)$) the number of preventive replacements (respectively corrective replacements and inspections) before t, $d_u(t)$ the accumulated unavailability duration of the system before t. Let us denote by T the length of a life-time cycle. The property of regeneration of the process $(X_t)_{t\geq 0}$ allows us to write:

$$C_\infty = \lim_{t\to\infty} \frac{\mathbb{E}(C(t))}{t} = \frac{\mathbb{E}(C(T))}{\mathbb{E}(T)}, \quad \text{where} \;\; C(t) = C_i N_i(t) + C_p N_p(t) + C_c N_c(t) + C_u d_u(t). \quad (5)$$

Let us set the “inspection scheduling function” introduced in [6] be constant then the threshold A_{sup} and the inter-inspection time Δt can be obtained by numerical minimisation of the cost criterion of the single-mode deteriorating system in mode M_1. The threshold A_{inf} corresponds to the optimal threshold for the single-mode deteriorating system in mode M_2. In this work, the cost criterion is optimised as a function of the parameter of the considered maintenance policy : the detection threshold h and the size of the sliding window m_a.

6 Numerical implementations

In this section, by using numerical implementations, the properties of the proposed maintenance policy are analysed. Throughout this section, the values of the maintenance costs are respectively $C_i = 5$, $C_p = 50$, $C_c = 100$ and $C_u = 250$. For the numerical implementations it is supposed that in the nominal mode M_1, $\alpha_1 = 1$ and $\beta_1 = 1$. Hence, the maintenance threshold A_{sup} is equal to 90.2 and $\Delta t = 4$. The previous values are the optimal values which minimise the long run maintenance cost for a single mode deteriorating system in mode M_1. To evaluate the maintenance policy, four different accelerated modes are considered. It means that for numerical implementations the real parameters of the second mode which are unknown for the observer are chosen among one of the four accelerated modes presented in table 1.

real second mode cases	1	2	3	4
α_2	2	1	2	1
β_2	1	3	2	7

Table 1. Real characteristic data of the second degradation mode.

The maintenance versus detection policy takes into account only the last $m_a - m'_a$ observations. In the maximum likelihood detection method the accelerated mode parameters α_2 and β_2 are estimated by using the last $m_a - m'_a$ available observations. For the numerical implementations it is supposed that $m_a - m'_a = 4$. It is supposed that the second mode is simulated as a gamma process with parameters chosen among one of the four considered accelerated modes presented in table 1. In order to be able to use any estimated parameter, the optimal preventive threshold corresponding to different second mode parameters $\alpha_2 \in [0, 1]$ and $\beta_2 \in [1, 10]$ are calculated. The given optimal maintenance thresholds correspond to the values which minimise the long run maintenance cost for a single mode deteriorating system in mode M_2 with $\Delta t = 4$. Once a change of mode is detected, the maintenance policy is adapted by using the value of optimal maintenance threshold corresponding to the estimated parameters α_2 and β_2. The aim is to propose a threshold h which leads to a low maintenance cost.

In this section, the problem of parameter optimisation for the considered maintenance policy is investigated. The parameter of interest is the detection threshold h.
The “optimal” value of h which leads to a minimal maintenance cost is numerically calculated. To define the optimal value of h, the maintenance cost is obtained for different values of h in the interval $[0, 15]$.

The value of h corresponding to the lowest maintenance cost is defined. To study the impact of the variation of the threshold h on the properties of the maintenance policy in addition to the maintenance cost for different values of h in the interval $[0, 15]$ the false alarm rate, the detection delay and the probability of preventive actions are calculated. These results are depicted in figure 1 for $\alpha_2 = 1$ and $\beta_2 = 3$. The choice of h it is not always based on the value which minimises the maintenance cost. For example, it is not sensible to take a value of h leading to the lowest maintenance cost if it corresponds to a false alarm rate close to 1. In this case, the maintenance policy is mostly a one threshold policy. As the detection algorithm has a high false alarm rate, when the system is deteriorating according to the nominal mode the detection algorithm detects an abrupt change where the estimated parameters are close to the nominal mode parameters. Therefore, the effective maintenance threshold is mostly A_{sup}. In this case the detection algorithm can not improve the quality of the maintenance policy in comparison to the one threshold policy. To avoid such results when a value of h leads to the lowest maintenance cost if it corresponds to a false alarm rate close to 1, we chose another value of h corresponding to an optimal balance between the maintenance cost and the false alarm rate. In order to give a global vision of the quality of the maintenance policy embedded an on-line change detection algorithm, in table 2 this policy is compared with the one threshold policy. The costs in table 2 correspond to the four different considered real cases of the accelerated mode M_2 defined in table 1.

case	Maximum likelihood	one preventive threshold
1	1.97	1.97
2	2.05	2.21
3	2.19	2.36
4	2.41	2.66

Table 2. Optimal costs corresponding to two maintenance decision rules.

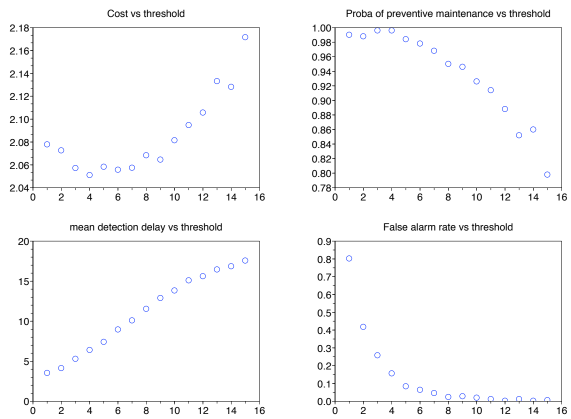

Fig. 1. The properties of the maintenance versus detection policy with $\alpha_2 = 1$, $\beta_2 = 3$.

It can be noticed that the use of on-line maximum likelihood detection method usually improves the quality of the maintenance policy in comparison with the one threshold maintenance policy (especially in cases 3 and 4). This is due to the fact that the efficiency of the detection method depends mostly on the Kullback-Leibler distance between the distribution of the two modes. In the cases 3 and 4 of table 1 this distance is very large but in the cases 1 and 2 the Kullback-Leibler distance is very small. It means that the two modes in the both cases 1 and 2 of the table 1 are very close and it is not easy to detect the abrupt change time.

7 Conclusion

In this paper the problem of condition based maintenance in the framework of gradually deteriorating systems in presence of abrupt change in the deterioration rate is addressed. The deterioration rate after the change is supposed to be unknown and it is estimated by a maximum likelihood method. The policy with on-line maximum likelihood detection method requires a great complexity of calculations. In this case, a study of the influence of the size of the sliding window on the maintenance cost is of great interest and it can be treated in future works. It is possible to go further and consider a totally unknown time dependent deterioration rate after the abrupt change and to propose an adpated maintenance versus detection policy.

References

M. Basseville and I.V. Nikiforov. *Detection of Abrupt Changes: Theory and Application*. Prentice.Hall, www.irisa.fr/sigma2/kniga, 1993.

C. Bérenguer, A. Grall, L. Dieulle, and M. Roussignol. Maintenance policy for a continuously monitored deteriorating system. *Probability in the Engineering and Informational Science*, 17(2):235–250, 2003.

L. Dieulle, C. Bérenguer, A. Grall, and M. Roussignol. Sequential condition-based maintenance scheduling for a deteriorating system. *EJOR*, 150(2):451–461, 2003.

M. Fouladirad and A. Grall. A maintenance decision rule with embedded bayesian on-line change detection for gradually deteriorating systems. *Journal of Risk and Reliability*, 222(3):359–369, 2008.

M. Fouladirad, A. Grall, and L. Dieulle. On the use of on-line detection for maintenance of gradually deteriorating systems. *Reliability Engeniering Sytem and Safety*, 93(12):1814–1820, 2008.

A. Grall, L. Dieulle, and C. Berenguer. Continuous- time predective maintenance scheduling for a detetiorating system. *IEEE transactions on reliability*, 51(2):141–150, 2002.

T. L. Lai. Information bounds and quick detection of parameter changes in stochastic systems. *IEEE Transactions on Information Theory*, 44:2917–2929, 1998.

T.L Lai. Sequential changepoint detection in quality control and dynamical systems. *Journal of Royal Statistical Society*, 57:613–658, 1995.

B. Saassouh, L. Dieulle, and A. Grall. Adaptive maintenance policy for a deteriorating system with random change of mode. page 5. ESREL 2005, 30 June 2005. Avec acte.

J.M. van Noortwijk. A survey of the application of gamma processes in maintenance. In *Reliability Engineering and System Safety*, 2007.

Hongzhou Wang. A survey of maintenance policies of deteriorating systems. *European Journal of Operational research*, 129:469–489, 2002.