Variations on CUSUM tests for flutter monitoring ⋆

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Abstract. Flutter is a critical aircraft instability phenomenon. One important issue to be handled online during flight testing is flutter monitoring, here addressed as a detection problem. Subspace detection algorithms have been designed for vibration-based monitoring. Several online flutter monitoring algorithms have been designed, based on a recursive version of the subspace residual and on the CUSUM test for detecting changes in a specific instability indicator w.r.t. a *fixed reference* modal parameter (identified on a safe structure), but are too conservative. Two solutions have been elaborated for overcoming that issue. The first one performs flutter detection w.r.t. a reference modal state *predicted* close to instability using the a priori knowledge of an aeroelastic model and/or experimental flight test data. The second solution is an adaptive flutter monitoring algorithm which is based on a *moving reference* version of the above and updates the reference modal parameter during the online test. The advantages and drawbacks of the different solutions are discussed based on experimental results obtained on simulation data.
Keywords. Aircraft instability, flutter monitoring, adaptive algorithms, CUSUM tests.

1 Introduction

An aircraft is a complex structure subject to vibration and aeroservoelastic forces. A critical aircraft instability phenomenon is known under the name of flutter. Flutter results from an unfavorable interaction of aerodynamic, elastic and inertial forces, and may cause major failures (Gero, 1999; Kehoe, 1995). Air worthiness regulations require high security standards for each new aircraft to prevent a destructive aeroelastic instability phenomenon known as flutter. Consequently, one important issue to be addressed on-line during the flight testing process is the flight flutter monitoring problem.

Modeling the aircraft with a linear state-space model, the aircraft dynamics can be summarized by the set of eigenvalues and eigenvectors of the state transition matrix, the so-called modes and mode-shapes. Based on data recorded from e.g. accelerometers under natural (turbulent) and non-measured excitation conditions, these parameters can be estimated using subspace identification (Mevel et al., 2006; Pickrel and White, 2003). Online in-flight flutter monitoring aims at the early detection of a deviation in the modal parameters before it develops into flutter. Change detection is a natural approach in this context. For a scalar instability criterion ψ and a critical value ψ_c, the idea is to test online between two hypotheses $\mathbf{H}_0$ and $\mathbf{H}_1$ about ψ, typically $\psi > \psi_c$ and $\psi \leq \psi_c$, for a stable and unstable aircraft, respectively.

Because of the unknown excitation, a likelihood ratio approach cannot be used. A residual built on the estimating function associated with subspace identification can be handled instead (Basseville et al., 2000). Thanks to the local approach, the residual is assumed to be Gaussian, and manifest the change from $\mathbf{H}_0$ to $\mathbf{H}_1$ as a change in its mean. A CUSUM test can be run as an approximation to the optimal test. Several algorithms of that type have been designed and investigated, based on different stability criteria ψ: decreasing damping coefficient (Mevel et al., 2005) or flutter margin (Zhou et al., 2007; Zouari et al., 2006), pairs of time-varying frequencies or damping coefficients (Basseville et al., 2006), mode shapes correlations (Zouari et al., 2007). From such a flutter detection, the flutter airspeed may be estimated. All these algorithms perform the online monitoring of deviations in a flutter indicator with respect to a *fixed reference* modal parameter (identified on a safe structure). The drawback of such an approach is that the resulting flutter detection then corresponds to a light trend of the indicator towards instability and thus the resulting estimated flutter airspeed is conservative.

Two techniques have been designed for overcoming that issue. The first one performs flutter detection with respect to a reference modal state *predicted* close to instability using the *a priori* knowledge of

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an aeroelastic model and/or experimental flight test data (Zouari et al., 2008a). The second one is an adaptive flutter monitoring algorithm based on a *moving reference* version of the above which updates the reference modal parameter during the online test (Zouari et al., 2008b). The purpose of this paper is to discuss the advantages and drawbacks of these algorithms based on results obtained on simulation data. In section 2, the subspace statistics and the CUSUM test for flutter monitoring are introduced. Different variations on the CUSUM test based on a fixed, a predicted or a moving reference are introduced in section 3. The experimental results are presented in section 4. Some conclusions are drawn in section 5.

2 Modal monitoring and CUSUM tests

Subspace-based residual. It is well known (Ewins, 2000) that vibration-based structural monitoring boils down to monitoring the eigenstructure of the state transition matrix F of a linear dynamic system:

$$\begin{cases} X_{k+1} = F\ X_k + V_{k+1} \\ Y_k = H\ X_k \end{cases} \tag{1}$$

namely the roots (λ, Φ_λ) of $\det(F - \lambda I) = 0, (F - \lambda I)\ \Phi_\lambda = 0$. Let $\varphi_\lambda \triangleq H\Phi_\lambda$, and $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\Phi \end{pmatrix}$, where Λ is the vector containing the λ's (called modes), Φ the matrix whose columns are the φ_λ's (called mode shapes), and vec the column stacking operator. A reference parameter $\theta_\star$ is assumed available, identified on data from the system in a reference state, using output-only covariance-driven subspace identification algorithm. It is based on the factorization of the covariances: $R_i \triangleq \mathbf{E}(Y_k Y_{k-i}^T) = HF^iG$ with $G \triangleq \mathbf{E}(X_k Y_k^T)$, and consists in computing the SVD of the empirical Hankel matrix $\widehat{\mathcal{H}}^\star_{p+1,q}$ filled with $\widehat{R}_i$'s. Based on the subspace interpretation of the SVD, the parameter $\theta_\star$ can be characterized by:

$$U(\theta_\star)^T\ \widehat{\mathcal{H}}^\star_{p+1,q} = 0 \tag{2}$$

where orthonormal matrix U is subject to:

$$U(\theta_\star)^T\ \mathcal{O}_{p+1}(\theta_\star) = 0 \tag{3}$$

and $\mathcal{O}_{p+1}(\theta)$ is the observability matrix in modal basis. Although not unique, matrix U can be treated as a function of θ. For detecting a change in θ w.r.t. $\theta_\star$, the solution in (Basseville et al., 2000) handles a statistics (residual) built on the parameter estimating function in (2):

$$\zeta_n(\theta_\star) \triangleq \sqrt{n}\ \text{vec}(U(\theta_\star)^T\ \widehat{\mathcal{H}}_{p+1,q}) \tag{4}$$

where $\widehat{\mathcal{H}}_{p+1,q}$ the empirical Hankel matrix for *new data* from the (possibly changed) system. Testing if $\theta = \theta_\star$ holds true – or equivalently deciding that $\zeta_n(\theta_\star)$ is zero – requires the generally unknown distribution of $\zeta_n(\theta_\star)$. A solution is to use the statistical local approach (Basseville and Nikiforov, 1993; Benveniste et al., 2006) and assume close hypotheses:

$$\widetilde{\mathbf{H}}_0 :\ \theta = \theta_\star \quad \text{and} \quad \widetilde{\mathbf{H}}_1 :\ \theta = \theta_\star + \Upsilon/\sqrt{n} \tag{5}$$

where vector Υ is unknown, but fixed. Let $\mathbf{E}_\theta$ be the expectation when the actual system parameter is θ, and define the mean deviation (Jacobian) and the covariance:

$$\mathcal{J}_n(\theta_\star, \theta) \triangleq 1/\sqrt{n}\quad \partial/\partial\tilde{\theta}\ \mathbf{E}_\theta\ \zeta_n(\tilde{\theta})\Big|_{\tilde{\theta}=\theta_\star}\ ,\ \ \Sigma_n(\theta_\star, \theta) \triangleq \mathbf{E}_\theta\left(\zeta_n(\theta_\star)\ \zeta_n(\theta_\star)^T\right) \tag{6}$$

Then, provided that $\Sigma_n(\theta_\star, \theta)$ is positive definite, and for all Υ, the residual ζ_n in (4) is asymptotically Gaussian distributed under both hypotheses in (5):

$$\text{when } n \to \infty, \quad \Sigma_n(\theta_\star, \theta)^{-1/2}\left(\zeta_n(\theta_\star) - \mathcal{J}_n(\theta_\star, \theta)\ \Upsilon\right) \ \longrightarrow\ \mathcal{N}(0, I) \tag{7}$$

Thus a deviation $\Upsilon \neq 0$ in the system parameter θ is reflected into a change in the mean of ζ_n. Consistent estimates of $\mathcal{J}_n(\theta_\star,\theta)$ and $\Sigma_n(\theta_\star,\theta)$, based on data samples recorded on the reference system, are given in (Basseville et al., 2000) and (Zhang and Basseville, 2003), respectively. Assume now that $\mathcal{J}_n(\theta_\star,\theta)$ is full column rank (f.c.r.). It may be preferable to handle the following normalized residual:

$$\overline{\zeta}_n(\theta_\star) \stackrel{\Delta}{=} \mathcal{K}_n(\theta_\star,\theta)\ \zeta_n(\theta_\star)\ ,\ \ \mathcal{K}_n(\theta_\star,\theta) \stackrel{\Delta}{=} \overline{\Sigma}_n(\theta_\star,\theta)^{-1/2}\ \mathcal{J}_n(\theta_\star,\theta)^T\ \Sigma_n(\theta_\star,\theta)^{-1} \tag{8}$$

where $\overline{\Sigma}_n(\theta_\star,\theta) \stackrel{\Delta}{=} \mathcal{J}_n(\theta_\star,\theta)^T\ \Sigma_n(\theta_\star,\theta)^{-1}\ \mathcal{J}_n(\theta_\star,\theta)$. From (7), $\overline{\zeta}_n(\theta_\star)$ is asymptotically Gaussian:

$$\text{when } n\to\infty,\quad (\overline{\zeta}_n(\theta_\star) - \overline{\Sigma}_n(\theta_\star,\theta)^{1/2}\ \Upsilon)\ \longrightarrow\ \mathcal{N}(0,I) \tag{9}$$

For an *on-line* detection algorithm, a data-driven computation for $\overline{\zeta}_n(\theta_\star)$ in (8) is preferable to the covariance-driven computation in (4). Assuming $n > p+q$ and introducing $\mathcal{Y}^{+T}_{k,p+1} \stackrel{\Delta}{=} (\,Y_k^T\ \ldots\ Y_{k+p}^T\,)$ and $\mathcal{Y}^{-T}_{k,q} \stackrel{\Delta}{=} (\,Y_k^T\ \ldots\ Y_{k-q+1}^T\,)$, the statistics (8) writes as the sum:

$$\overline{\zeta}_n(\theta_\star) = \sum_{k=q}^{n-p} Z_k(\theta_\star)/\sqrt{n}\ ,\ \text{ where }\ Z_k(\theta_\star) \stackrel{\Delta}{=} \mathcal{K}_n(\theta_\star,\theta)\ \text{vec}\left(U(\theta_\star)^T\ \mathcal{Y}^{+}_{k,p+1}\ \mathcal{Y}^{-T}_{k,q}\right) \tag{10}$$

From (9) and (10), $\sum_{k=q}^{n-p} Z_k(\theta_\star)/\sqrt{n}$ is asymptotically Gaussian distributed, with mean zero under $\widetilde{\mathbf{H}}_0$ and $\overline{\Sigma}(\theta_\star,\theta)^{1/2}\ \Upsilon$ under $\widetilde{\mathbf{H}}_1$. The arguments in (Benveniste et al., 1990)[5.4.1] lead to another approximation: for n large enough, and $k=1,\ldots,n$, one can regard $Z_k(\theta_\star)$ itself as if it was i.i.d. and Gaussian, with mean 0 if no change in θ occurred before time k, and with a non-zero mean after a change occurred.

Those properties hold true whatever $\mathcal{J}_n$ in (8) and (10) is. For monitoring any function $\psi(\theta)$, one should replace $\mathcal{J}_n(\theta_\star,\theta)$ in (8) with $\mathcal{J}_n(\theta_\star,\theta)\ \mathcal{J}^\star_{\theta\psi}$, where $\mathcal{J}^\star_{\theta\psi} = \partial\theta/\partial\psi|_{\theta=\theta_\star}$.

CUSUM test for monitoring a scalar instability indicator. That a *scalar* ψ crosses a critical threshold ψ_c is reflected into a change *of the same sign* in the mean ν of the independent Gaussian variables $Z_k(\theta_\star)$. The CUSUM test may be used for testing $\mathbf{H}_0\!:\nu>0$ against $\mathbf{H}_1\!:\nu\leq 0$. A relevant procedure when neither the current actual hypothesis nor the sign and magnitude of the change in ψ are known consists in:
i) Setting a *minimum change magnitude* $\nu_m>0$, and testing between $\mathbf{H}_0\!:\nu>\nu_m/2$ and $\mathbf{H}_1\!:\nu\leq-\nu_m/2$:

$$S_n(\theta_\star) \stackrel{\Delta}{=} \sum_{k=q}^{n-p} (Z_k(\theta_\star)+\nu_m)\ ,\ \ T_n(\theta_\star) \stackrel{\Delta}{=} \max_{k=q,\ldots,n-p} S_k(\theta_\star)\ ,\ \ g_n(\theta_\star) \stackrel{\Delta}{=} T_n(\theta_\star) - S_n(\theta_\star) \underset{\mathbf{H}_0}{\overset{\mathbf{H}_1}{\gtrless}} \varrho \tag{11}$$

ii) Running two tests in parallel, for a decreasing and an increasing ψ, respectively;
iii) Making a decision from the first test which fires;
iv) Resetting all sums and extrema to zero and switching to the other one afterwards.
The choices of ν_m and ϱ are generally well decoupled (Basseville and Nikiforov, 1993)[Chap.10,11].

3 Variations on the CUSUM test

For detecting instability precursors with the CUSUM test (10)-(11), it is necessary to select:
a) An instability criterion ψ and a critical value ψ_c;
b) A reference state for the system, for identifying (or computing) $\theta_\star$ and/or computing $U(\theta_\star)$ in (3);
c) Estimators of the $\mathcal{J}_n(\theta_\star,\theta)$ and $\Sigma_n(\theta_\star,\theta)$ matrices in (6);
d) A minimum change magnitude ν_m and a threshold ϱ.

Three different solutions for *b)-c)* are reported and compared here:

1. $\theta_\star \stackrel{\Delta}{=} \theta_0$ *identified on reference data* for the *stable* system, and $\mathcal{J}_n, \Sigma_n$ estimated once for all on those data, namely: $\mathcal{J}_n(\theta_\star,\theta) \stackrel{\Delta}{=} \mathcal{J}(\theta_0)$ and $\Sigma_n(\theta_\star,\theta) \stackrel{\Delta}{=} \Sigma(\theta_0)$;
2. $\theta_\star \stackrel{\Delta}{=} \theta_c$, a critical parameter *computed* using both the reference θ_0 and an aeroelastic model, and $\mathcal{J}_n, \Sigma_n$ estimated recursively with the *test data* using an algorithm as in (Kailath and Sayed, 1999);
3. An adaptive $U(\theta_\star)$ computed with the *test data*, and $\mathcal{J}_n, \Sigma_n$ estimated recursively as in **2.**
Solution **1** was studied with different ψ (Basseville et al., 2006; Mevel et al., 2005; Zhou et al., 2007; Zouari et al., 2006; Zouari et al., 2007). Solutions **1** and **2** run with the flutter margin have been compared in (Basseville et al., 2007). The three solutions run with a damping coefficient are compared in section 4.

Solution **2.** The critical θ_c is computed using flutter prediction based on extrapolating the coefficients of the characteristic polynomial associated with the quasi-steady aeroelastic model $M\ddot{q} + (D + VB)\dot{q} + (K + V^2C)q = 0$, where q is the generalized coordinates vector, V the airspeed, M, D, K are the inertial, damping and stiffness matrices, B, C the aerodynamic damping and stiffness matrices (De Troyer et al., 2008). Assuming the system with order $2m$, the identification of those coefficients should be done in at least (2m+1) flight points for predicting the eigenvalues at instability. Solution **2** consists of:
i) Estimating the critical eigenvalues λ_c at flight point t using identified modal signature $(\theta_1, \ldots, \theta_t)$;
ii) Building the critical modal signature θ_c from λ_c and the mode-shapes φ_λ identified at flight point t;
iii) Using θ_c to compute the recursive residual $Z_k(\theta_c)$ in (10);
iv) Running the CUSUM test in (11) for flutter detection between flight points t and $t+1$;
v) Repeating these steps for flight point $t+1$: modal identification θ_{t+1} to update the prediction of θ_c and running the CUSUM test between $t+1$ and $t+2$.

Solution **3.** The left kernel U is estimated from a Hankel matrix built with L samples using (2): $\widehat{U}_n^T \sum_{k=n+q-L-\tau}^{n-p-\tau} \mathcal{Y}_{k,p+1}^{+} \mathcal{Y}_{k,q}^{-T} = 0$. The recursive residual in (10) is computed after a lag of τ samples and writes: $Z_n(\theta) \triangleq \mathcal{J}_n(\theta)^T \Sigma_n^{-1} \mathrm{vec}(\widehat{U}_n^T \mathcal{Y}_{n,p+1}^{+} \mathcal{Y}_{n,q}^{-T})$. The recursive estimates $\widehat{\mathcal{J}}_n, \widehat{\Sigma}_n$ of $\mathcal{J}_n, \Sigma_n$ are given in (Zouari et al., 2008b). Solution **3** consists in the following steps:
i) For an initial airspeed, modal identification is performed to estimate a fixed reference θ_0 and compute the fixed terms in $\widehat{\mathcal{J}}_n$. The data sample size L, lag τ, sample block size K, minimum magnitude of change ν_m, and threshold ϱ are chosen. $\widehat{\Sigma}_{L+\tau}^{-1}$ and $\widehat{\mathcal{J}}_{L+\tau}$ are computed with the first $L+\tau$ samples. The estimate $\widehat{U}_{L+\tau}$ is computed with $(Y_1, \ldots, Y_L)$ and used with $\widehat{\Sigma}_{L+\tau}^{-1}$ and $\widehat{\mathcal{J}}_{L+\tau}$ to compute $S_{L+\tau}$ in (11);
ii) Recursive loop: running the CUSUM test in (11). For each sample $n \geq L+\tau$, $\widehat{U}_n$ is computed with $(Y_{n-\tau-L}, \ldots, Y_{n-\tau})$ and used with $\widehat{\Sigma}_n^{-1}$ and $\widehat{\mathcal{J}}_n$ to compute S_n and g_n until $g_n \geq \varrho$. During this process, $\widehat{\Sigma}_n^{-1}$ and $\widehat{\mathcal{J}}_n$ are updated every K samples.

4 Example

The Hancock wing model. Data are simulated using the aeroelastic model of a rigid rectangular wing with constant chord allowing two degrees-of-freedom in bending and torsion (Hancock et al., 1985). Matrix F in (1) is expressed as a function of the airspeed V and the corresponding modal frequencies and damping coefficients are plotted in Figure 1. The typical bending-torsion coupling behavior of the flutter can be observed when frequencies get closer to each other and the damping coefficients move apart. The flutter onset can be estimated when the torsional damping coefficient reaches zero at the airspeed $V \simeq 88.5\ m/s$. An aircraft acceleration is simulated with sampling frequency $F_s = 50$ Hz and transition phase from $V = 20$ to $88\ m/s$ (close to flutter). In this speed range, 1000 samples (20 seconds) are simulated every $1\ m/s$ step to obtain two-dimensional time series with sample size $N = 69000$.

Numerical results. The parameters of the CUSUM test are $\nu_m = 0.1$ and $\varrho = 100$. For Solution **1**, the $U, \mathcal{J}, \Sigma$ matrices are computed once for the reference θ_0 at $V = 20\ m/s$ on a large data set and remain constant. The CUSUM test is applied for each mode to detect the decrease in the damping coefficient. For Solution **1** in Figure 2, except for some noise induced perturbations, the test values for the bending mode (Left) remain small while a flutter alarm is launched by the test for the torsional mode (Right). This is coherent with the damping coefficient behavior in Figure 4(a). It can be also checked that the alarm is raised approximately at the airspeed where the damping coefficient crosses its reference value at $V = 20\ m/s$ from above. That confirms the precision of the proposed test in monitoring flutter criteria w.r.t. a reference. However, the flutter airspeed estimate associated with that alarm is $V = 65\ m/s$ which can be considered conservative as compared to the true flutter airspeed.

The results obtained with Solution **2** are displayed on Figures 4 and 5 and compared with Solution **1**. Figure 4(a) shows θ_c predicted after t flight points and the typical coupling as expected. The flutter onset can be predicted when the second mode decreases to zero at $V \simeq 88\ m/s$. The modal signature θ_c is then estimated close to flutter at $V = 85\ m/s$. In Figure 4(b), the test values for flight data from $V = 40\ m/s$

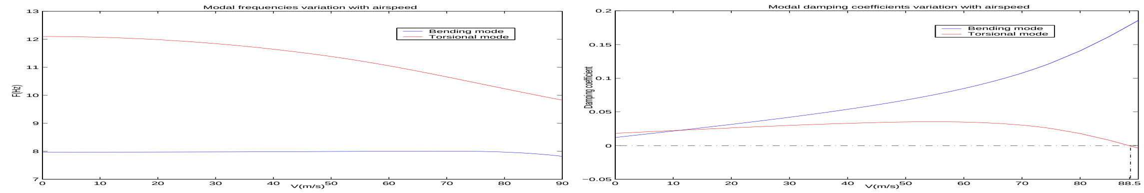


Fig. 1. Frequencies (Left) and damping coefficients (Right) of the bending (Blue) and torsional (Red) modes.

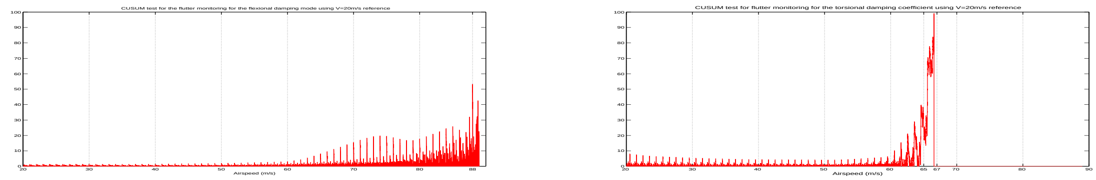


Fig. 2. Solution **1** with θ_0 at V=20 m/s. Bending (Left) and torsional (Right) modes.

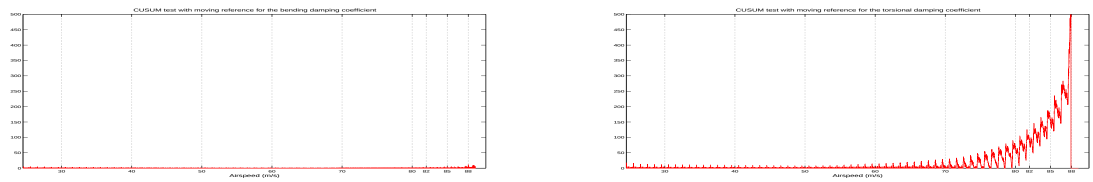
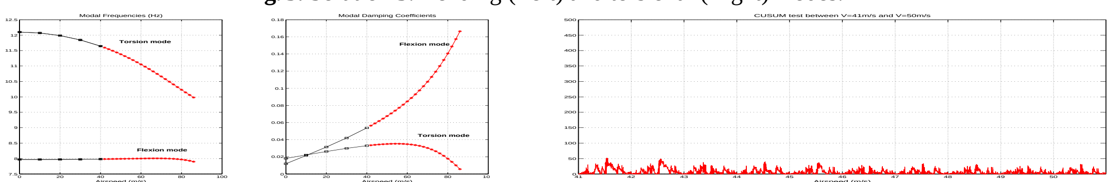
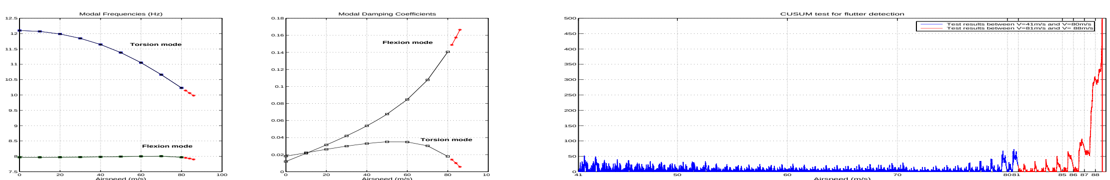


Fig. 3. Solution **3**. Bending (Left) and torsional (Right) modes.



(a) (b)

Fig. 4. (a) θ_c predicted after t flight points. (b) Solution **2** between flight points t and $t+1$.



(a) (b)

Fig. 5. (a) θ_c predicted at the current flight point. (b) Solution **1** (Blue), Solution **2** from current to next flight points (Red).

to the next flight point at $V = 50\ m/s$ remain small which confirms the stability of this flight region. Test results at the critical flight point are plotted in Figure 5. The flutter onset is detected from the second mode test at $V = 85\ m/s$ which confirms the predicted modal evolution.

Solution **3** is run with $L = 2000, \tau = 1000, K = 50$. The test values for the torsional mode in Figure 3 (Right) are also coherent with the modal behavior. The improvement concerns the detection closer to the flutter zone and provides a flutter airspeed estimate at $V = 78\ m/s$. This is more realistic than Solution **1** and that gives a larger flight envelope for the aircraft. Even though no abrupt torsional damping drop is observed at that airspeed in Figure 4 (Right), the decreasing rate becomes important and leads to flutter. It can also be noted in Figure 3 (Right) that the test values for light damping decrease within the airspeed range preceding the alarm remains small thanks to the update of U.

5 Discussion and conclusion

In this paper, we have described three online detection solutions for flutter monitoring. Solution **1** is based on a model-free subspace statistics, a local approach, and a CUSUM test. Solution **2** requires both an aeroelastic model for modal reference prediction and a recursive computation of the covariance matrix. Moreover, solution **3** updates the reference modal parameter online without model knowledge. The simulation results shown for a small structure suggest the improvement of Solutions **2** and **3** over Solution **1**. In particular Solution **2** improves the flutter airspeed estimation. Moreover, while prediction methods are able to determine flutter airspeed, the CUSUM test turns out useful to validate and improve flutter prediction results because: *i/* Flutter prediction based on modal identification is generally biased due to unmodeled dynamics (Lind, 2003); *ii/* The modal behavior close to flutter is generally well characterized, especially for modes related to flutter. The robustness of this approach w.r.t. errors in the modal parameters is illustrated with these experiments because the test results are not affected when using the mode-shapes φ_λ identified at the last flight point t instead of the critical mode-shapes φ_{λ_c}. However a limitation should be mentioned, which is the cost and availability of a flutter prediction model. As for Solution **3**, the experimental results show a significant improvement in the flutter airspeed estimation, despite this method does not detect the flutter itself but a brutal drop in the monitored parameter. Future investigations will consider more complex aircraft models. Another major issue is related to the actual dimension of the parameter vector θ to be monitored and the high number of correlated (scalar) instability criteria to handle at a time.

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