Delay time in monitoring jump changes in linear models

Marie Hušková[1], Zuzana Prášková[2], and Josef Steinebach[3]

[1] Department of Probability and Mathematical Statistics,
Charles University
Sokolovská 83, Prague, Czech Republic
huskova@karlin.mff.cuni.cz

[2] Department of Probability and Mathematical Statistics,
Charles University
Sokolovská 83, Prague, Czech Republic
praskova@karlin.mff.cuni.cz

[3] Mathematisches Institut, Universität zu Köln
Weyertal 86–90, D–50 931 Köln, Germany
jost@math.uni-koeln.de

Abstract. In this contribution we study the limiting distribution of the stopping time of a sequential procedure for monitoring jump changes in linear models. Our main result shows that stopping times based on weighted ordinary residuals have an asymptotic normal distribution, when the size of a training sample tends to infinity.

1 Introduction

In this paper, we investigate the limiting behaviour of a stopping time τ_m (as $m \to \infty$), which has been studied by Hušková and Koubková (2005) as a sequential procedure for monitoring jump changes in linear models. Such procedures are motivated by a wide range of applications, e.g., in economics and finance, bio- and geosciences, quality control or intensive care in medicine, to mention just a few. In contrast to earlier work of Chu et al. (1996), Horváth et al. (2004), Aue et al. (2006), who discussed sequential CUSUM type test statistics based on the sums of *ordinary* residuals, the procedure of Hušková and Koubková (2005) makes use of *weighted ordinary* residuals and is able to detect alternatives which cannot be detected otherwise (confer the simulation results in Hušková and Koubková, 2005, Section 3).

We assume that the data follow the linear regression model

$$Y_i = \boldsymbol{X}_i^T \boldsymbol{\beta}_i + e_i\,, \quad 1 \le i < \infty, \tag{1}$$

with possible changes in the p-dimensional regression parameters $\boldsymbol{\beta}_i\,,\ 1 \le i < \infty$. Stability of the historical data is requested by the so-called *noncontamination condition*

$$\boldsymbol{\beta}_1 \;=\; \ldots \;=\; \boldsymbol{\beta}_m.$$

The observations $Y_1, \ldots, Y_m$ represent the training period (historical data), $\{\boldsymbol{X}_i\,,\ 1 \le i < \infty\}$ is a sequence of p-dimensional regression vectors (random or nonrandom), and $\{e_i,\ 1 \le i < \infty\}$ are the random errors. It is assumed that the data are arriving sequentially.

Detection of a change in the linear model is formulated as a sequential hypothesis testing problem, where the null hypothesis H_0 corresponds to the model without any change, i.e.,

$$H_0:\ \boldsymbol{\beta}_i = \boldsymbol{\beta}_0\,, \quad 1 \le i < \infty,$$

and the alternative hypothesis H_A reflects that the model changes at some unknown time-point, that is,

$$H_A: \text{there exists } k^* \ge 1 \text{ such that } \boldsymbol{\beta}_i = \boldsymbol{\beta}_0\,,\ 1 \le i < m + k^*, \text{ but}$$
$$\boldsymbol{\beta}_i = \boldsymbol{\beta}_0 + \boldsymbol{\Delta}_m,\ m + k^* \le i < \infty, \quad \boldsymbol{\Delta}_m \ne \mathbf{0},$$

where $\boldsymbol{\beta}_0\,, \boldsymbol{\Delta}_m$ and k^* are unknown parameters.

In the following we assume that $\boldsymbol{\Delta}_m$ changes with m (typically $\boldsymbol{\Delta}_m \to \mathbf{0}$ at some rate as $m \to \infty$, see details below).

For the detection of changes in the above model, Chu et al. (1996) developed sequential procedures under the paradigm that data can be observed cheaply (at no cost) and that there is a training period of size m, which can be used for calibration of the model with the goal of an on-line monitoring of the data afterwards. To this end, they suggested CUSUM type test statistics calculated from recursive residuals as well as a fluctuation test based on differences between estimates of the regression coefficients. Their approach has been generalized and extended in various directions. Leisch et al. (2000), for example, suggested a so-called generalized fluctuation test, whereas Zeileis et al. (2005) developed MOSUM type test statistics based on observations taken from a moving window over the data.

CUSUM type test statistics based on ordinary and on recursive residuals have further been investigated in Horváth et al. (2004) assuming independent, identically distributed (i.i.d.) errors, while Aue et al. (2006) generalized this setting to allow for a large class of dependent errors. Berkes et al. (2004) discussed similar problems for the change detection of GARCH(p, q) processes, and Hušková et al. (2007, 2008a) studied the testing of stability in autoregressive time series (see also Hušková and Koubková (2006)).

In what follows we are interested in deriving the limiting distribution of the stopping time in the Hušková and Koubková (2005) sequential procedure. Such results, to the best of our knowledge, have been initiated by Aue (2004) and Aue and Horváth (2004), who considered a "change in the mean" model, and they have further been investigated by Kvesic (2006) and Aue et al. (2008, 2009) for various CUSUM type test statistics in linear models. For a recent result concerning MOSUM type sequential procedures confer also Horváth et al. (2008).

Our main result below extends the above works and proves the asymptotic normality of the delay time of CUSUM type test statistics for monitoring jump changes in linear models, based on weighted residuals.

2 Main results

We investigate the limiting distribution of the stopping time τ_m defined as follows:

$$\tau_m = \inf\{k \geq 1 : \, Q(m,k) \geq c\, q_\gamma^2(k/m)\}, \tag{2}$$

with $\inf \emptyset := +\infty$. Here, $Q(m,k)$ are CUSUM type test statistics (detectors) based on the observations $Y_1, \ldots, Y_{m+k}$, $k = 1, 2, \ldots$, the function $q(t)$, $t \in (0, \infty)$, is a (critical) boundary function, and the constant $c = c(\alpha)$ is chosen such that, for $\alpha \in (0,1)$ fixed,

$$\lim_{m\to\infty} P_{H_0}\big(\tau_m < \infty\big) \;=\; \alpha, \qquad \lim_{m\to\infty} P_{H_A}\big(\tau_m < \infty\big) \;=\; 1. \tag{3}$$

For the sake of convenience we introduce the notation

$$\widehat{e}_i \;=\; Y_i - \boldsymbol{X}_i^T \widehat{\boldsymbol{\beta}}_m\,, \tag{4}$$

where $\widehat{\boldsymbol{\beta}}_m$ is the least squares estimator of the regression parameter $\boldsymbol{\beta} = \boldsymbol{\beta}_0$, based on the first m observations, i.e.,

$$\widehat{\boldsymbol{\beta}}_m = \left(\sum_{i=1}^m \boldsymbol{X}_i \boldsymbol{X}_i^T\right)^{-1} \sum_{i=1}^m \boldsymbol{X}_i Y_i. \tag{5}$$

Moreover,

$$V(m,k) \;=\; \Big(\sum_{i=m+1}^{m+k} \boldsymbol{X}_i \widehat{e}_i\Big)^T \boldsymbol{C}_m^{-1} \Big(\sum_{i=m+1}^{m+k} \boldsymbol{X}_i \widehat{e}_i\Big), \tag{6}$$

$$\boldsymbol{C}_k = \sum_{i=1}^k \boldsymbol{X}_i \boldsymbol{X}_i^T, \quad k = 1, 2, \ldots, \tag{7}$$

and the detector, based on $Y_1, \ldots, Y_{m+k}$, is chosen as

$$Q(m,k) \;=\; V(m,k)/\widehat{\sigma}_m^2\,, \tag{8}$$

where $\widehat{\sigma}_m^2$ is an estimator of σ^2, calculated from $Y_1, \ldots, Y_m$, such that

$$\widehat{\sigma}_m^2 - \sigma^2 = O_P(m^{-\beta}), \qquad \beta \geq \frac{1-2\gamma}{4(1-\gamma)}. \tag{9}$$

Hušková and Koubková (2005) provided an approximation $c = \widehat{c}_p(\alpha,\gamma)$ of the critical value c via the limiting behaviour of the stopping rule under the “no change” null hypothesis and under the following assumptions:

(A.1) $\{e_i\}_{i=1}^{\infty}$ is a sequence of independent, identically distributed (i.i.d.) random variables such that $\mathsf{E}\, e_1 = 0,\ 0 < \mathsf{Var}\, e_1 = \sigma^2 < \infty$ and $\mathsf{E}\,|e_1|^{\nu} < \infty$ for some $\nu > 2$;

(A.2) $\{\boldsymbol{X}_i^T\}_{i=1}^{\infty}$ is a strictly stationary sequence of p-dimensional vectors $\boldsymbol{X}_i^T = (1, X_{2i}, \ldots, X_{pi})$, which is independent of $\{e_i\}_{i=1}^{\infty}$;

(A.3) there exist a positive definite matrix $\boldsymbol{C}$ and a constant $0 < \eta < 1$ such that

$$\max_{1\le k\le m} \Big| \frac{1}{k}\sum_{i=1}^{m} \boldsymbol{X}_i \boldsymbol{X}_i^T - \boldsymbol{C} \Big| k^{1-\eta} \;=\; \mathcal{O}_P(1),$$

where $|\cdot|$ denotes the maximum norm of vectors and matrices.

The boundary functions q has been chosen from the following class of functions:

(B.1) $q(t) = q_\gamma(t) = (1+t)\,(t/(t+1))^{\gamma},\ t \in (0,\infty)$, where γ is a tuning constant taking values from the interval $\big[0, \min\{\frac{1}{2}, 1-\eta\}\big)$.

Here we recall the main result of Hušková and Koubková (2005).

Theorem 1. *Let $Y_1, Y_2, \ldots$ follow the model* (1) *and let assumptions* (A.1) – (A.3) *and* (B.1) *be satisfied. Then, under H_0 , for any $x \in \mathbb{R}$,*

$$\lim_{m\to\infty} P\left(\sup_{1\le k<\infty} \frac{V(m,k)}{\widehat{\sigma}_m^2 q_\gamma^2(k/m)} \le x \right) \;=\; P\left(\sup_{0<t\le 1} \frac{\sum_{i=1}^{p} W_i^2(t)}{t^{2\gamma}} \le x \right), \tag{10}$$

where $\{W_i(t);\ 0 \le t \le 1\}$, $i = 1, \ldots, p$, are independent standard Wiener processes.

Hušková and Koubková (2006) also established the limiting result (10) in model (1) with $\boldsymbol{X}_i = (Y_{i-1}, \ldots, Y_{i-p})^T$, i.e., in case that the observations Y_i follow an autoregressive process (see also Example 4 below). Moreover, combining the proof of Theorem 1 in Hušková and Koubková (2005) and Lemma 4.5 in Hušková at al. (2007), we can prove (10) under model (1), in which $\{\boldsymbol{X}_i e_i\}$ is a martingale difference sequence. See, e.g., Examples 1–3 in the next section.

The main theorem of our paper is proved under the following assumptions, in which we assume that $m \to \infty$:

(C.1) $k^* \;=\; \mathcal{O}(m^{\theta})$ with $0 < \theta < \frac{1-2\gamma}{2(1-\gamma)}$ (early changes);

(C.2) $\sum_{i=1}^{m} \boldsymbol{X}_i e_i \;=\; \mathcal{O}_P(\sqrt{m})$;

(C.3) $\max_{1\le k\le K_m} \left\{|\sum_{i=m+1}^{m+k} \boldsymbol{X}_i e_i|^2/k^{2\gamma}\right\} \;=\; \mathcal{O}_P(K_m^{1-2\gamma})$ for any $K_m = o(m)$, and
$\max_{k_m\le k\le K_m} \left\{|\sum_{i=m+k+1}^{m+K_m} \boldsymbol{X}_i e_i|^2\right\} \;=\; \mathcal{O}_P(K_m - k_m)$ for any $K_m = o(m)$ with $K_m - k_m \to \infty$;

(C.4) there are positive definite (symmetric) matrices $\boldsymbol{C}$, $\boldsymbol{C}^*$ and a constant $\eta \ge \frac{1}{2}$ such that

$$|\boldsymbol{C}_m - m\boldsymbol{C}|/m^\eta \;=\; \mathcal{O}_P(1)$$

$$\max_{k^*\le k\le K_m} |\boldsymbol{C}_{m+k} - \boldsymbol{C}_m - k\boldsymbol{C}^*|/k^\eta \;=\; \mathcal{O}_P(1)$$

for any $K_m = o(m)$, but $K_m/k^* \to \infty$;

(C.5) it holds that

$$m\,|\boldsymbol{\Delta}_m|^2 \to \infty, \quad \text{but} \quad m^{\frac{(1-2\gamma)(2\eta-1)}{3-2\gamma-2\eta}}\,|\boldsymbol{\Delta}_m|^2 \to 0$$

where $\boldsymbol{\Delta}_m = \boldsymbol{\beta}_{m+k^*} - \boldsymbol{\beta}_0$;

(C.6) for any $K_m = o(m)$, but $K_m/k^* \to \infty$, $\left(\sum_{i=m+1}^{m+K_m} \boldsymbol{X}_i e_i\right)^T \boldsymbol{C}^{-1}\boldsymbol{C}^*\boldsymbol{\Delta}_m$ has an asymptotic normal distribution with mean zero and variance $v_m^2 = \sigma^2 K_m \boldsymbol{\Delta}_m^T \boldsymbol{C}^*\boldsymbol{C}^{-1}\boldsymbol{C}^*\boldsymbol{C}^{-1}\boldsymbol{C}^*\boldsymbol{\Delta}_m$, i.e.

$$v_m^{-1}\Big(\sum_{i=m+1}^{m+K_m} \boldsymbol{X}_i e_i\Big)^T \boldsymbol{C}^{-1}\boldsymbol{C}^*\boldsymbol{\Delta}_m \xrightarrow{D} N(0,1),$$

where $N(0,1)$ is a standard normal random variable.

Theorem 2. *Let $Y_1, Y_2, \ldots$ follow the model* (1) *and let assumptions* (C.1) – (C.6) *and* (B.1) *be satisfied. Then, under H_A, for any $x \in \mathbb{R}$,*

$$\lim_{m\to\infty} P\big(\tau_m \le a_m + d_m\sqrt{a_m}\,x\big) \;=\; \Phi(x), \tag{11}$$

where

$$a_m = m\Big(\frac{c\,\sigma^2}{m\boldsymbol{\Delta}_m^T \boldsymbol{C}^*\boldsymbol{C}^{-1}\boldsymbol{C}^*\boldsymbol{\Delta}_m}\Big)^{1/(2(1-\gamma))},$$

$$d_m = \frac{\sigma}{1-\gamma}\,\frac{\sqrt{\boldsymbol{\Delta}_m^T \boldsymbol{C}^*\boldsymbol{C}^{-1}\boldsymbol{C}^*\boldsymbol{C}^{-1}\boldsymbol{C}^*\boldsymbol{\Delta}_m}}{\boldsymbol{\Delta}_m^T \boldsymbol{C}^*\boldsymbol{C}^{-1}\boldsymbol{C}^*\boldsymbol{\Delta}_m},$$

and Φ denotes the distribution function of a standard $N(0,1)$ random variable.

Proof. The proof is based on the fact that under (C.1) – (C.6) and (B.1),

$$P\big(\tau_m \le a_m + \sqrt{a_m}\,d_m\,x\big) = P\Big(\max_{1\le k\le K_m} \frac{V(m,k)}{\widehat{\sigma}_m^2\, q_\gamma^2(k/m)} \ge c\Big), \tag{12}$$

where $K_m = K(x,m) = [a_m + \sqrt{a_m}\,d_m\,x]$, with $[\,\cdot\,]$ denoting the integer part, and on a proper decomposition of the statistic $V(m,k)$. We confer to Hušková et al. (2008b) for many details.

Remark 1. The assertion of Theorem 2 remains true if σ^2 is replaced by an estimator $\widehat{\sigma}_m^2$ satisfying assumption (9).

3 Examples

There are many models satisfying the assumptions of Theorem 2. Here is a list of some examples.

Example 1. Let $\{\boldsymbol{X}_i\}_{i=1}^{\infty}$ and $\{e_i\}_{i=1}^{\infty}$ be independent sequences such that

(i) $\boldsymbol{X}_i = (1, X_{i2}, \ldots, X_{ip})^T = (1, \tilde{\boldsymbol{X}}_i^T)^T$, where $\{\tilde{\boldsymbol{X}}_i^T\}$ are i.i.d random vectors with finite covariance matrix $\boldsymbol{V}$ and such that $\mathsf{E}\,|\tilde{\boldsymbol{X}}_1|^4 < \infty$;

(ii) $\{e_i\}_{i=1}^{\infty}$ is a martingale difference sequence, with respect to (w.r.t.) the filtration $\{\mathcal{F}_i\}$, where $\mathcal{F}_i = \sigma\{e_t, t \leq i\}$, such that $\mathsf{E}\,e_i^2 = \sigma^2$, $\mathsf{E}\,|e_i|^{\nu} \leq K < \infty$, for some constants $\nu > 2$ and $K > 0$, and $m^{-1}\sum_{i=1}^{m} \mathsf{E}\,(e_i^2|\mathcal{F}_{i-1}) \xrightarrow{P} \sigma^2$ as $m \to \infty$. [This includes the i.i.d. case as well as ARCH and GARCH-type stationary sequences.]

In this case, Condition (C.2) follows from the Chebyshev inequality, (C.3) from the Hájek-Rényi inequality for martingale differences (see, e.g., Hušková et al. (2007), Lemma 4.3). Condition (C.4) holds, with $\boldsymbol{C} = \boldsymbol{C}^* = \mathsf{E}\,\boldsymbol{X}_1\boldsymbol{X}_1^T$, as a consequence of the strong law of large numbers (SLLN) for i.i.d. random vectors and of the Hájek-Rényi inequality, and (C.6) holds as a consequence of the central limit theorem (CLT) for martingales. Relation (9) holds with $\widehat{\sigma}_m^2 = \frac{1}{m}\sum_{i=1}^{m}\widehat{e}_i^2$.

The assertion even remains true, if the $\{\tilde{\boldsymbol{X}}_i\}$ are independent, but not necessarily identically distributed, with zero mean and $\mathsf{E}|\tilde{\boldsymbol{X}}_i|^4 \leq D < \infty$ for all i and some $D > 0$, and if, for some k_{0m} such that $k_{0m}\, m^{-(1-2\gamma)/(2(1-\gamma))}|\boldsymbol{\Delta}_m|^{1/(1-\gamma)} \to 0$ as $m \to \infty$, $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_{m+k_{0m}}$ are i.i.d., with $\mathsf{E}\,\boldsymbol{X}_1\boldsymbol{X}_1^T = \boldsymbol{C}$ positive definite, and $\boldsymbol{X}_{m+k_{0m}+1}, \boldsymbol{X}_{m+k_{0m}+2}, \ldots$ are i.i.d., with $\mathsf{E}\,\boldsymbol{X}_{m+k_{0m}+1}\boldsymbol{X}_{m+k_{0m}+1}^T = \boldsymbol{C}^*$ positive definite.

Example 2. Let $\{\boldsymbol{X}_i\}$ and $\{e_i\}$ be independent sequences such that

(i) $\boldsymbol{X}_i = \left(1, \tilde{\boldsymbol{X}}_i^T\right)^T$, where $\widetilde{X}_{ij} = \sum_{k=0}^{\infty}\alpha_k(j)v_{i-k}$, $i = 1, 2, \ldots, j = 2, \ldots, p$ (i.e., $\widetilde{X}_{ij}$ is a linear process, e.g. an ARMA process) with $\{v_i\}$ being i.i.d. random variables with zero mean and finite fourth moment, and $\alpha_k(j) = c_j\alpha^k$, for some constants c_j and $0 < \alpha < 1$ (or $\alpha_k(j) = \mathcal{O}(\alpha^k)$ as $m \to \infty$, for some $0 < \alpha < 1$, uniformly in j);

(ii) $\{e_i\}_{i=1}^{\infty}$ is a martingale difference sequence, that satisfies the same conditions as given in Example 1.

Then $\{\boldsymbol{X}_i e_i\}$ are martingale differences w.r.t. the filtration $\{\mathcal{G}_i\}$, $\mathcal{G}_i = \sigma\{\boldsymbol{X}_t e_t, t \leq i\}$, which implies that Conditions (C.2), (C.3), and (C.6) hold true, with $\boldsymbol{C} = \boldsymbol{C}^* = \mathsf{E}\,\boldsymbol{X}_1\boldsymbol{X}_1^T$. Moreover, the first property in (C.4) follows from the SLLN and martingale properties of linear processes (see Hall and Heyde (1980)). The second property in (C.4) is a consequence of the Hájek-Rényi type inequality given in Kokoszka and Leipus (1998). The variance estimator $\widehat{\sigma}_m^2$ can be chosen as in Example 1.

Example 3. Let $\{\boldsymbol{X}_i\}$ and $\{e_i\}$ be independent sequences such that

(i) $\boldsymbol{X}_i = (1, \tilde{\boldsymbol{X}}_i^T)^T$, where $\{\tilde{\boldsymbol{X}}_i\}$ is a sequence of stationary, strongly mixing random vectors of size $-4r/(r-2)$, $r > 2$, with $\mathsf{E}|\tilde{\boldsymbol{X}}_1|^{r+\delta} \leq M < \infty$ for some $\delta > 0$ and $M > 0$;

(ii) $\{e_i\}$ is a martingale difference sequence as given in Example 1.

Again, Conditions (C.2), (C.3), (C.4), and (C.6) hold true, with $\boldsymbol{C} = \boldsymbol{C}^* = \mathsf{E}\,\boldsymbol{X}_1\boldsymbol{X}_1^T$. The first part of Condition (C.4) follows from Hall and Heyde (1980) by using the same arguments as in Example 2, while the second one is a consequence of the Hájek-Rényi inequality for the strong mixing sequences $\{X_{ij}X_{ik} - \mathsf{E}\,X_{ij}X_{ik}\}_{i=1}^{\infty}$ $(j, k = 2, \ldots, p)$, which is a modification of results by Bai and Perron (1998), Lemma A.6, and Qu and Perron (2007) together with the supplement to the latter paper (Lemma A.1 there). The variance estimator $\widehat{\sigma}_m^2$ can again be chosen as in Example 1.

Example 4. Let $\{Y_i\}$ be a sequence that satisfies model (1) where

(i) $\boldsymbol{X}_i = (Y_{i-1}, Y_{i-2}, \ldots, Y_{i-p})^T$;

(ii) $\{e_i\}$ are i.i.d. random variables with zero mean, variance $\sigma^2 > 0$ and $\mathsf{E}\,|e_i|^\nu < \infty$ for some $\nu > 2$;
(iii) the parameters $\boldsymbol{\beta}_i$ satisfy

$$\boldsymbol{\beta}_i = \boldsymbol{\beta}_0\,, \qquad 1 \le i < m + k^*,$$
$$\boldsymbol{\beta}_i = \boldsymbol{\beta}_0 + \boldsymbol{\Delta}_m\,, \quad m + k^* \le i < \infty,\ \boldsymbol{\Delta}_m \ne 0,$$

where $\beta_p^0 \ne 0$, and all the roots of the polynomial $z^p - \beta_1^0\, z^{p-1} - \ldots - \beta_p^0$ are inside the unit circle.

Thus,

$$Y_i = \beta_1^0\, Y_{i-1} + \ldots + \beta_p^0\, Y_{i-p} + e_i\,, \qquad 1 \le i < m + k^*,$$
$$Y_i = (\beta_1^0 + \Delta_1^m)\, Y_{i-1} + \ldots + (\beta_p^0 + \Delta_p^m)\, Y_{i-p} + e_i\,, \qquad m + k^* \le i < \infty.$$

Then $\{\boldsymbol{X}_i e_i\}$ is a martingale difference sequence w.r.t. the filtration $\{\mathcal{F}_i\}, \mathcal{F}_i = \sigma\{e_t, t \le i\}$. Hence, Condition (C.2) is satisfied as a consequence of Chebyshev's inequality, (C.3) follows from Lemma 4.3 in Hušková et al. (2007), Condition (C.4) remains true, with $\boldsymbol{C} = \boldsymbol{C}^* = \mathsf{E}\,\boldsymbol{X}_m\boldsymbol{X}_m^T$, for any $\frac{3}{4} < \eta < 1$ and any $K_m = \mathcal{O}(m^{(1-2\gamma)/(2(1-\gamma))}|\boldsymbol{\Delta}_m|^{-1/(1-\gamma)})$, see Hušková et al. (2008b) for details. Condition (C.6) holds as a consequence of a CLT for martingale differences. The variance σ^2 can be replaced by $\widehat{\sigma}_m^2 = \frac{1}{m}\sum_{i=1}^m \widehat{e}_i^2$.

Acknowledgement. The work of Hušková and Prášková was supported by grants GAČR 201/09/J006 and MSM 0021620839.

References

Aue, A. (2004). *Sequential Change-Point Analysis Based on Invariance Principles*, PhD thesis, University of Cologne.

Aue, A., and Horváth, L. (2004). Delay time in sequential detection of change. *Statistics and Probability Letters*, **67**, 221–231.

Aue, A., Horváth, L., Hušková, M., and Kokoszka, P. (2006). Change-point monitoring in linear models. *Econometrics Journal*, **9**, 373–403.

Aue, A., Horváth, L., Kokoszka, P., and Steinebach, J. (2008). Monitoring shifts in mean: Asymptotic normality of stopping times. *Test*, **17**, 515–530.

Aue, A., Horváth, L., and Reimherr, M.L. (2009). Delay times of sequential procedures for multiple time series regression models. *Journal of Econometrics*, to appear.

Bai, J., and Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, **66**, 47–78.

Berkes, I., Gombay, E., Horváth, L., and Kokoszka, P. (2004). Sequential change-point detection in GARCH(p, q) models. *Econometric Theory*, **20**, 1140–1167.

Chu, C.-S., Stinchcombe, M., and White, H. (1996). Monitoring structural change, *Econometrica*, **64**, 1045–1065.

Hall, P., and Heyde, C.C. (1980). *Martingale Limit Theory and Its Application.* Academic Press, New York.

Horváth, L., Hušková, M., Kokoszka, P., and Steinebach, J. (2004). Monitoring changes in linear models. *Journal of Statistical Planning and Inference*, **126**, 225–251.

Horváth, L., Kühn, M., and Steinebach, J. (2008). On the performance of the fluctuation test for structural change, *Sequential Analysis*, **27**, 126–140.

Hušková, M., and Koubková, A. (2005). Monitoring jump changes in linear models. *Journal of Statistical Research*, **39**, 59–78.

Hušková, M., and Koubková, A. (2006). Sequential procedures for detection of changes in autoregressive sequences. In: *Proceedings of Prague Stochastics 2006*, (M. Hušková, and M. Janžura, Eds). MATFYZPRESS, Charles University Prague, 437–447.

Hušková, M., Kirch, C., Prášková, Z., and Steinebach, J. (2008a). On the detection of changes in autoregressive time series, II. Resampling procedures. *Journal of Statistical Planning and Inference*, **138**, 1697–1721.

Hušková, M., Prášková, Z., and Steinebach, J. (2007). On the detection of changes in autoregressive time series, I. Asymptotics, *Journal of Statistical Planning and Inference*, **137**, 1243–1259.

Hušková, M., Prášková, Z., Steinebach, J. and Černí ková, A. (2008b). Delay time in monitoring jump changes in linear models. Preprint, Charles University, Prague.

Kokoszka, P., and Leipus, R. (1998). Change-point in the mean of dependent observations, *Statistics and Probability Letters*, **40**, 385–393.

Koubková, A. (2006). Sequential change-point detection in linear models. In: *Proceedings of Prague Stochastics 2006*, (M. Hušková, and M. Janžura, Eds). MATFYZPRESS, Charles University Prague, 484–494.

Kvesic, M. (2006). *Strukturanalyse linearer Modelle*, Diploma thesis, University of Cologne.

Leisch, F., Hornik, K., and Kuan, C.-M. (2000). Monitoring structural changes with the generalized fluctuation test, *Econometric Theory*, **16**, 835–854.

Qu, Z., and Perron, P. (2007). Estimating and testing structural changes in multivariate regressions. *Econometrica*, **75**, 459–502.

Zeileis, A., Leisch, F., Kleiber, C., and Hornik, K. (2005). Monitoring structural change in dynamic econometric models. *Journal of Applied Econometrics*, **20**, 99–121.